E-Simplicity versus e-Complexity : Some Remarks on the Perspectives of the Universal Design for Learning

Svatoslav Ondra
Masaryk University
Brno, Czech Republic
ondra@teiresias.muni.cz

Petr Peňáz
Masaryk University
Brno, Czech Republic
penaz@teiresias.muni.cz

Abstract

Mathematics was developed to approach the complexity of nature through the simplicity of numbers – thus it can be considered as the first digitization process in history. Nevertheless, to reach its goal, mathematics itself develops more and more complex structures which are, in terms of their readability and accessibility, as complicated as nature itself, using two or even more dimensions in writing and encoding formulae. The same development can be observed in informatics and the media used for presenting scientific formulae. The lecture analyzes and compares existing technologies and methods of adjustment applied in the accessibility of didactical and scientific documents. Based on their experience in equal opportunities within the European academic settings, the authors consider the question of whether that variety of approaches really helps to make the academic curricula accessible.

1 Making mathematics accessible as an intellectual challenge

Making mathematics accessible for the blind has presented a challenge several times in the history of mankind. It was a fashionable issue in European courts of the Enlightenment and this Enlightenment sponsoring is also behind the creation of Valentin Haüy's and Louise Braille's studies from the end of the 18th and the beginning of the 19th centuries. It revived in various national contexts each time the Braille notation was adopted and adapted to particular national purpose at the end of the 19th and the beginning of the 20th centuries. A massive spread of information technologies in the 1990s brought the last wave. As you can see in this introductory reflection, we are focusing on the European context of the mentioned issue as our experience is based on fifteen years of providing service to students with visual impairment in the Central European region. For this reason, we apologize that we are leaving Arabic, Indian, Chinese and other notations aside and we are only considering them to the extent of their immediate influence on the European tradition.

The last accessibility wave is characterized by a certain simplification that we would like to overcome: mathematical notation is commonly assumed as an intuitively familiar and given fact: it is assumed that the mathematical notation simply exists and the task is not to find what it is or what it is not, but to make it accessible. In our opinion, this approach leads to some misunderstandings and before proceeding towards an analysis of the common accessibility practice, let us consider, what is actually supposed to become accessible?

2 The development of mathematical notation

2.1 Ancient Greece – plain textual linearity

The contemporary mathematical notation is a varied mixture of diverse elements and notation techniques which has been created for over millennia but has existed as a whole for a surprisingly short time, if it is a whole at all. Mathematicians of ancient Greece who are commonly viewed as creators of the European mathematical tradition, did not use any specific notation. Given the fact that classical Greek had no diacritics and used letters for numerical values, the notation was purely linear alphabetical and its theoretical accessibility for the blind is only complicated by the rich graphics illustrating geometrical phenomena.[1]

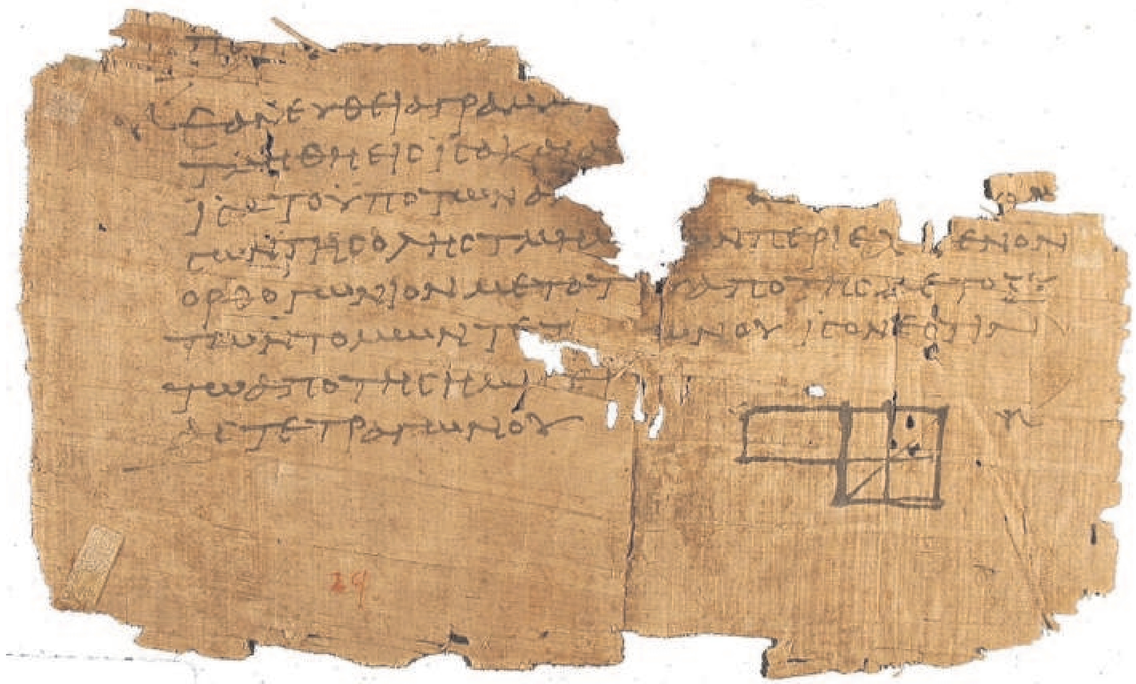

Figure 1: Euclid's Elements – Papyrus Oxyrynchus (P. Oxy I 29), dated 75-125 AD.

In the Hellenistic period at the beginning of the Christian era, however, diacritical marks begin to emerge (to indicate accents and other phonetic phenomena) as well as tachygraphic symbols (including indexes abbreviating stable combinations of characters), so the notation of common texts as such ceases to be linear.[2]

2.2 Diophantus of Alexandria and early symbols

Diophantus of Alexandria generously used the elements that had complicated the orthography of common Greek texts in the 3rd century A.D. In his *Arithmetica*, there is a specific symbol for the unknown value, tachygraphic abbreviations expressing squares and cubes etc. The aim of these elements is visual: to fit a larger amount of information into the reader's field of vision. In mathematics, this care for the field of vision is more important than in a literary text: as it provides a navigation system among arguments. It allows the sight to follow the key pieces of information and compare them, without reading the stereotypical and formal ones. This makes it possible to perform mathematical operations with the mere use of sight, while the older method presumed memorizing all elements, which are included in a considered relation. It is precisely because he takes into consideration the specifics of mathematical work that Diophantus adds a specific technique which has no counterpart in Greek literary texts. He solves operations with fractions by a planar arrangement of numbers: he writes the denominator above

[1] File: *P. Oxy. I 29.jpg – Wikimedia Commons* [online]. 7. October 1994. [cit. 2012-01-08]. Available at <http://en.wikipedia.org/w/index.php?title=File:P._Oxy._I_29.jpg&oldid=426463448>

[2] File: The Bodleian Library, University of Oxford. *Euclid – Elementa, spread 9* [database online]. ©2004 Octavo. [cit. 2012-01-08]. Available at <http://www.rarebookroom.org/Control/eucmsd/index.html>.

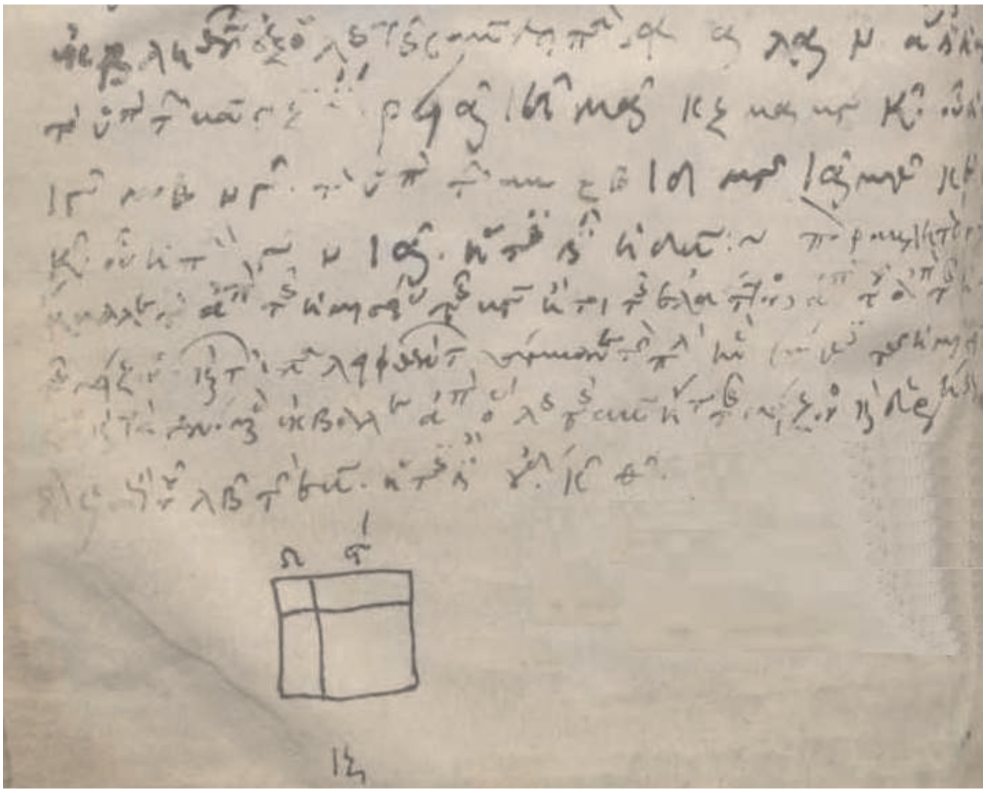

Figure 2: Euclid's Elementa. Byzantine manuscript dated 888 AD.

the numerator (without a line as fraction separator).

It still holds for Diophantus that his mathematical notation in principle remains – in contrast to contemporary use – identical to a common text, even to such extent that it determines his methods. When we consider the development of mathematical notation from a blind person's perspective, it is very important to notice the difference between Diophantus's technique and the common one today, because it is illustrative of some of the differences between the work of the sighted and the blind. [1] [2]

In chapter I, 28 of his *Arithmetica*, Diophantus assigns the following task: ***Find two numbers whose sum and the sum of whose squares are given numbers.***[3]

2.2.1 Contemporary solution conditioned by algebraic symbolism

Considering that Diophantus only conceives of a solution in the set of positive rational numbers, a conventional solution to his task using the algebraic apparatus common today, which became stable in Europe between the Renaissance and the Baroque, looks as follows:

The task: *Find positive rational numbers* x, y *such that* $x+y=a$, $x^2+y^2=b$, *where* a, b *are positive rational numbers.*

Solution:

(1) $x+y=a$

(2) $x^2+y^2=b$

[3] Εὑρεῖν δύο ἀριθμοὺς ὅπως καὶ ἡ σύνθεσις αὐτῶν καὶ ἡ σύνθεσις τῶν ἀπ' αὐτῶν τετραγώνων ποιῇ δοθέντας ἀριθμούς.

(3) Let $x = s + \frac{a}{2}$.

Thus, by substitution into (1):

$s + \frac{a}{2} + y = a$, follows that:

(4) $y = \frac{a}{2} - s$

Then, by substitution into (2):

$\left(s + \frac{a}{2}\right)^2 + \left(\frac{a}{2} - s\right)^2 = b$, follows that:

$$s^2 + as + \frac{a^2}{4} + s^2 - as + \frac{a^2}{4} = b$$

$$s^2 = \frac{2b-a^2}{4}$$

(5) $s = \frac{\sqrt{2b-a^2}}{2}$

Result for x by substitution into (3):

$$x = \frac{\sqrt{2b-a^2}}{2} + \frac{a}{2} = \frac{a+\sqrt{2b-a^2}}{2}$$

Result for y by substitution into (4):

$$y = \frac{a}{2} - \frac{\sqrt{2b-a^2}}{2} = \frac{a-\sqrt{2b-a^2}}{2}$$

Condition of solubility: $\sqrt{2b-a^2}$ must be a positive rational number.

2.2.2 Period solution based on verbal description

The level of generality commonly used today is not realistic for Diophantus because his algebra is very elementary – he uses one unknown s, he is also able to express its powers s^2, $s^3 n$, but here his possibilities end. It is interesting to note the individual steps of Diophantus's mathematical reasoning based on simple verbal description:

1. Diophantus specifies the conditions of solubility in the first step in a specifying statement without revealing the way that led him to this proposition:
   Twice the sum of the squares minus the square of the sum of the two numbers must be a square. This is a necessary condition.[4]
2. Then he chooses concrete numbers that meet the condition of solubility and for which the task should be solved. The whole subsequent solution is demonstrated on these concrete numbers, not universally:
   Let the sum of the sought-after numbers give the value of 20 *and the sum of their squares give the value of* 208.[5]

[4] Δεῖ δὴ τοὺς δὶς ἀπ’ αὐτῶν τετραγώνους τοῦ ἀπὸ συναμφοτέρου αὐτῶν τετραγώνου ὑπερέχειν τετραγώνῳ. ἔστι δὲ καὶ τοῦτο πλασματικόν.

[5] Ἐπιτετάχθω τὴν μὲν σύνθεσιν αὐτῶν ποιεῖν M^o κ′, τὴν δὲ σύνθεσιν τῶν ἀπ’ αὐτῶν τετραγώνων ποιεῖν M^o ση′.

3. Then Diophantus applies algebraic elements he innovatively introduced himself to solve the first equation. Yet his solution is for the most part based on operations which are not further described and are performed in his mind:
   Let us denote the difference of the two sought-after numbers by $2s$*. Then, the bigger number will have the value* s *plus the value of* 10 *(a half of the sum of the numbers), while the smaller will have the value* $10 - s$*. Thus the resulting sum of the sought-after numbers has the value of* 20*, their difference being* $2s$*.*[6]
4. When solving the second equation, the amount of memory and imagination needed is even higher:
   What remains is that the sum of the squares of the sought-after numbers produce the value of 208*. However, their sum produces* $2s^2$ *plus the value of* 200*, which in sum equals the value of* 208*, and for* s *the resulting value is* 2*.*[7]
5. The final application of the result of the solved equation is easy to follow from today's perspective:
   Back to the task: the bigger sought-after number is the value 12*, the smaller number the value* 8*. And this meets the requirements.*[8]

From this short analysis of Diophantus it is evident that a Greek mathematical text lost its linear simplicity not in relation to the needs of mathematics, but due to a development of textual notation as such. On the other hand, this nascent complexity offered methods which had been unthinkable before.

2.3 Notations of European Middle Ages

In the Middle Ages, it became a custom to only write down words by their root or initial, while grammatical parts of words, endings (with frequent words the whole of them) were usually written with hundreds of ligatures (tachygraphic marks) added to the root. This principle applied in Europe to both Latin and Greek scripts until the arrival of printing and even after:[9]

Only a small part of the richness of hand-written marks has entered the contemporary conventions for literary texts written in the Latin alphabet: ligatures for the letter combinations **ff**, **fi**, **fl**, ligature **&**, exponential marks for ordinal numbers 1ª, 2º in Roman languages etc. In comparison to common texts, mathematical documents seem to be a much more conservative environment as they have preserved much more of these techniques otherwise extinct.

2.4 Fibonacci's digitization

In the field of mathematics, the Middle Ages pushed forward the work with planar, i.e. non-linear, arrangement. Leonardo Fibonacci in his work Liber Abaci (1202) introduced Indian-Arabic symbols for numbers and a positional number system called modus Indorum. This introduced a brand new principle into a notation which enables mechanical performance of arithmetic operations with bigger numbers. This way, another part of mathematical operations passed from the memory to the visual domain. However, the introduction of the positional decimal system brought at the same time a complication: it is not possible to read numbers from left to right only, because the real meaning of the symbols situated at the left of the sequence is only apparent after counting the orders, which requires to proceed from right to left.

[6] Τετάχθω δὴ ἡ ὑπεροχὴ αὐτῶν s β′. Καὶ ἔστω ὁ μείζων s α′ καὶ M^{o} ι′, τῶν ἡμίσεων πάλιν τοῦ συνθέματος, ὁ δὲ ἐλάσσων M^{o} ι′ ⋏ s α′. καὶ μένει πάλιν τὸ μὲν σύνθεμα αὐτῶν M^{o} κ′, ἡ δὲ ὑπεροχὴ s β′.

[7] Λοιπόν ἐστιν καὶ τὸ σύνθεμα τῶν ἀπ' αὐτῶν τετραγώνων ποιεῖν M^{o} ση′· ἀλλὰ τὸ σύνθεμα τῶν ἀπ' αὐτῶν τετραγώνων ποιεῖ Δ^{Υ} β′ M^{o} ς′. Ταῦτα ἴσα M^{o} ση′ καὶ γίνεται ὁ s M^{o} β′.

[8] Ἐπὶ τὰς ὑποστάσεις. ἔσται ὁ μὲν μείζων M^{o} ιβ′, ὁ δὲ ἐλάσσων M^{o} η′. καὶ ποιοῦσι τὰ τῆς προτάσεως.

[9] File: Euclid Lueneburg ms page 8.jpg – Wikimedia Commons [online]. 11 February 2008. [cit. 2012–01–08]. Available at <http://commons.wikimedia.org/w/index.php?title=File:Euclid_Lueneburg_ms_page_8.jpg&oldid=26558035>

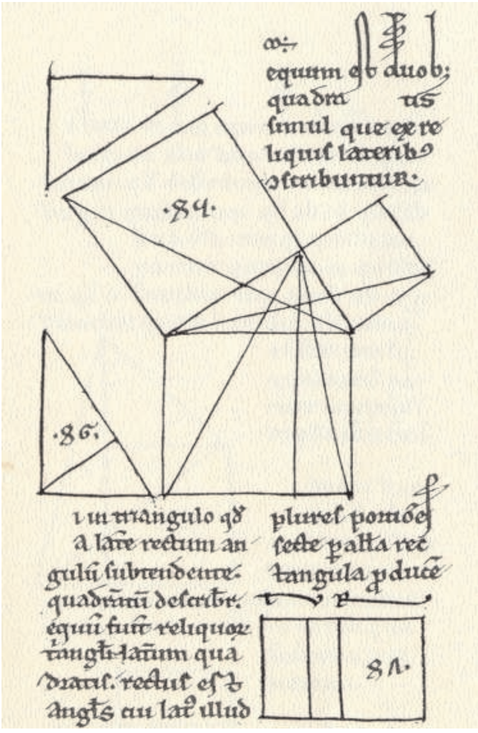

Figure 3: Euclid's Elements – Manuscript from Lüneburg, fragment. AD 1200.

2.5 Emergence of other mathematical symbols

Fibonacci was also the first to apply the line as graphical separator in fractions, although this only became systematically used by the Czech-German mathematician Johannes Widmann in 1489. Widmann is the first to consistently apply symbols for addition and subtraction which had only existed in a non-systematic way till then (+ is in fact a tachygraphic mark for Latin et, related to the symbols &, ð). It is not the purpose of this paper to describe or comment on individual symbols. However, to illustrate the complexity of what is called mathematical notation, let us mention at least the basic principles and symbols.

Multiplication was trying to find its symbol much longer and in fact, it has not found it: the cross (×) was introduced by William Oughtred in 1631, while simultaneously a square (□) and an asterisk (*) were in use; at the end of the 17th century, Leibniz replaced the cross with a dot (·) due to the possible confusion of the cross with the algebraic x. All these symbols are still being used, not always exactly differentiated. The case of the symbol for division is by no means simpler: Leibniz replaced the slash with a colon in the 18th century, while the Swiss Jonann Rahn with the obelus (÷). Again, the necessary unity has never come.

The decimal mark for separating the fractional part of a number from its integral part has not reached unity either: the original bar above the fractional part was replaced with a vertical line, later reduced to a decimal comma or a decimal point in the Anglo-Saxon region (originally, with different position in Britain and the USA) and when we add a different solution in countries writing Arabic, the map of the decimal mark is as varied as the map of the electrical socket standards.

Brackets of various types were gradually asserted from the 16th century onwards and became quite stable with Leibniz and Euler. A consistent indexical marking of powers is a matter of the developments between Descartes (1637) and Newton.

Between the 16th and 18th centuries, a completely new element of the mathematical notattion developed: stylized letters used as graphical elements separating groups of planarily arranged group of symbols: r ($\sqrt{}$) for the root, $\int$ for integral, Σ for sum, and so on. The level of stylization is such that a common user does not perceive these symbols as letters. The mark for absolute value comes from as late as 1841, from Karl Weiserstraß, while other symbols, for example in the logic and the sets, were added in the 20th century.

In total, it is apparent that the system of mathematical marks can be called a system only with restraint: it is an open set of not fully stable and originally very individual habits that are also integrally diverse. From the functional perspective, it is a combination of semantically ambiguous formal elements (indexation as a sign of power) with semantically unambiguous functional elements (root mark); infix notation (arithmetical operations) is combined with prefix notation (logarithms, sums) and so on. Despite the overall aims of internationality and universality, it is a very strange conservative world preserving more historical elements than common conventional notations of literary texts. A common mathematical notation of the early 21st century consists of the following formal elements:

1. common alphabetical characters, used as a text or in a rich variety of abbreviations
2. specific (non-alphabetical) symbols
3. indexes (based on both alphabetical and specific signs)
4. stylized letters used as graphical elements,
5. organizing (navigating) graphical elements
6. pictures.

2.6 The graphics

The European mathematics, as follows from the Egyptian tradition, is initially interested rather in geometrical solutions, for which graphics is as necessary as for geography. Thus, the mathematical manuscripts are full of graphics, while the commenting notation is a plain linear text. Researchers like Diophantus or later Fibonacci started concentrating on numerical solutions and thus the whole development is towards digitization, or rather arithmetization and algebraization. As of the 20th century, there are no geometrical phenomena impossible to express and solve numerically. The whole trend culminates in computerization, which is initially purely alphanumerical (and thus appears blind friendly) and incompatible with mathematical notation in its conventional form.

This led to specific text editor choices among mathematicians, such as ChiWriter [4] by Cay Horstmann (and to a much lower degree, WordPerfect) from 1986. ChiWriter was user friendly precisely in the area of mathematical notation, and so it resisted even the competition of TEX[5] by Donald Knuth till the 1990s, although TEXreached its version 3.0 containing MEGAFONT already in 1989. During the 1990s, ChiWriter definitely gave way to TEXwhen its development terminated in 1996 and no other system has gained a comparable popularity for mathematical purposes, although all widely spread editors (WordPerfect 5.1, MS Word, Mac Pages and the like) include modules for mathematical notation. TEXbecame the unofficial mathematicians' standard at the turn of the century also because of its open source code, although it is not a de facto standard and work with this editor is incomparable with WYSIWYG editors as far as user comfort is concerned.

However, the last decade of the 20th century with its mass IT popularity and commercialization brought a radical turn in the development: the need to assert information technologies in all social strata gradually turned the attention towards multimedia technologies. Audio and video began to replace text, thus making virtual digital environment similarly inaccessible for the blind as the real physical world. A large amount of software with a professional graphical output (Maple, Mathematica, Maple, Geometrix, Archimedes and so on) has emerged for mathematical purposes, but this does not mean a retreat of the TEXas a main tool.

Notwithstanding the information technology, graphics has always been a converter between a numerical solution and the real world of objects with spatial relations among these objects, and mathematics was created to solve these relations and has not ceased to be used to do so. Human mind is based on visual perception of objects and numerical solutions are secondary to this perception. Graphical depiction of objects and their relations thus never ceased to serve as explanations of the more abstract numerical solutions. This naturally leads to the question whether it is useful to employ graphics in mathematical education of the blind. As it follows from the argument below, it indeed turns out to be necessary.

3 Creation and development of tactile notation

Shifting from notations for the sighted to those for the blind, it is interesting to note that in many details, the history of the script for the blind is a repetition of what happened before and after in the development of the visual script. From the very beginning, these systems are characterized by the typical dilemma between a compatibility with the mainstream visual notation and the specificity of touch. At the turn of the 19th century, this dilemma showed in the struggle between Valentin Haüy's (and his followers' in Institut national des jeunes aveugles) tactile Latin alphabet and Louis Braille's raised dots. Braille himself was aware of this problematic dichotomy (which he himself created when he supported the raised dots) and he strived for the development of both systems. In his view, tactile graphics served the blind to address the sighted.

3.1 Systems based on Braille

3.1.1 Six-dot notation of literary texts

Later, the dilemma showed in the issue of whether the spreading Braille systems serve to transcribe visual scripts or they are independent. Braille's concept contains the potential for both: on the one hand, Braille rejected phonetic code of Barbier and he insisted on the conventional orthography, on the other hand he initially did not deal with the difference between the upper and lower case, did not include the Anglo-Saxon w etc., which made his system simpler than the visual writing. From 1837, the Braille system includes prefixes, i.e. it uses symbols with dots 4, 5, and 6 as switches multiplying the basic set of 64 symbols. Combinations with these specific prefixes (or even strings of prefixes) are the systemic elements that differentiate Braille systems in the proper sense from tactile representations of a visual encoding (like programming languages, TEX etc.).

Beginning from the international congress which in 1878 marked Braille as the primary communication system for the blind, the Braille system gradually developed to meet the needs of hundreds of languages independently of the particular visual alphabets used in respective regions (see World Braille Usage [6]). Creation of these codes was accompanied by processes similar to those when visual orthographies were established, especially the tachygraphic contractions. National systems thus vary in whether contractions are considered an integral part of the system (e.g. in German, where certain combinations of characters are contracted – diphthongs, digraphs, trigraphs) or whether it is only an option for a specific purpose (e.g. English Grade 2, Grade 3, while Grade 1 remains one of the possible standards), or where contractions do not exist at all (e.g. Czech). Preservation of contracted spelling in Braille systems is illustrative of the conservativism of the system: it is a principle which the visual script has only kept in mathematical and scientific notations. The English Grade 2 and Grade 3 furthermore represent (more than German contractions) cases of an orthographic independence of the tactile writing.

3.1.2 Six-dot notation of scientific texts

The main problem of Braille, however, rarely concerns the coverage of the standard orthography of a given language (at least not in European writing). Among much more complicated tasks there are transcription rules for foreign characters (in proper names or borrowings), specific typographic features, and symbols for scientific texts. There are only a few of these transcription systems (unlike transcription tables for common writing). In the mid-20th century, it was only the following systems whose influence reached beyond their local environment:

1. French Braille Code[10] [8]
2. Marburg system [9], [10]
3. Nemeth Braille Code[11] [11], [12], [13]

In 1970s and 1980s, another three large international systems were added:

4. Russian system [14]
5. British system[12] [15]
6. Spanish system [16]

Towards the end of the 20th century, standards for transcription exist in many countries – some adopt one of the above-mentioned systems or they depend on them to various degrees (for example, the Chinese system [17] on Marburg). In practice, these codes have two different aims: a) codification of specific characters and symbols, b) codification of rules for writing documents in science, i.e. standards for rendering features encoded in visual writing by graphics or in a visually specific way (sometimes called transcription rules as they are mostly used for transcription of existing visual documents).

In the coexistence of these systems, the old struggle between an integrating solution and the Braille's independence continues: they differ from one another in the level of dependence on visual notation (in this context, the independence of the Nemeth code is well known).

There has been a remarkable initiative in the establishment of the committee for a unification of the Braille notation in English-speaking countries, which resulted in Unified English Braille (2004) [7], completely different from the Nemeth code. It is very illustrative that this initiative, despite its name and thirteen years of development, has not resulted into unification but has become a new standard; first, for four countries (Australia, New Zealand, South Africa, and Nigeria) and from 2010 for Canada, too.

A coexistence of such diverse systems is not surprising for those who know the history of scientific notation of the sighted as it is an exact copy of the situation which existed in Europe in the late Baroque and early Classical periods: academic texts were following the main national and international authorities while the differences between these authorities did not seem surprising, nor was achieving unity stated as a goal. It very accurately corresponds with the mentioned historical situation that besides national and supranational authorities, there are many more individual solutions, so in practice it is often the case that a norm is only applied to publishing of printed materials for national or international distribution, while it is not an individual's goal to actively learn and use it for his own purposes. This disintegration of the system was largely assisted by the integrative education policy: mainstream teachers cannot follow or access the Braille conventions so they do not see a reason why exert pressure to respect them comparable in a blind person's life to pressures exerted on a sighted person. Each understandable solution usually tends to be accepted, which again resembles Europe of the early Baroque period.

[10] France, Madagascar, Portugal.

[11] Canada, Israel, Lebanon, New Zealand, Pakistan, Saudi Arabia, Singapore, Sri Lanka, Thailand, USA, Western Samoa; Greek adaptation.

[12] UK, Ireland, Australia, Bahrain, Hong Kong, Keyna, Jordan, Nigeria, Saudi Arabia, Sierra Leone, Zimbabwe.

3.1.3 Eight-dot notation of scientific texts

In the 1980s, information technology enters the field of tactile writing, quite complex as it already was, and with it, the eight-dot braille of tactile displays. It is paradoxical of this situation that although the computer becomes a common reality during the 1990s and there are national norms for the six-dot braille (sometimes more than one norm), systematic eight-dot norms are very exceptional:

1. Stuttgarter 8-Punkt-Mathematikschrift [18]
2. GS Braille (the eight dot variant) [19]
3. National eight-dot notations of the Lambda code [20]

As we can see from this list, these activities were taking place at the end of the 20th and the beginning of the 21st centuries. The majority of users (as there were or still are no standards in most languages) are not using any of the above systems and usually follow personal habits, based on the native codes of application or hardware in use and the national six-dot standards. There are usually no specific transcription rules (in terms of 3.1.2), but the authors try to make it similar (as much as possible) to the existing six-dot standard or some typical computer input (in terms of 3.2).

3.2 Notations based on typical computer inputs

In the 1990s, i.e. before the emergence of real eight-dot norms for the Braille notation of scientific texts, a completely new and, given the period, logical idea appears in the tactile writing of mathematics: to use those writing systems whose use was enforced to the sighted users by information technologies. In fact, this is a continuation of the old struggle between compatibility with the main stream (visual notation) and specificity of touch: Braille systems in the proper sense of the term (based on tactile-logical combinations) compete with systems that copy the visual solution. Be it inputs for compilers or for text editors, both procedures assume the use of the keyboard for writing a linear string (alphanumerical with the limited number of specific symbols accessible from the keyboard). The above-mentioned TEXsystem has even led to such consequences that many users follow visually a linear version of mathematical structures all the way up to the final compilation, just like a programmer, and this has explicit psychological consequences for the behavior of the population: more and more people recognize such coding as acceptable for sight. The best known tactile solutions based on this principle are:

1. ASCII-Mathematikschrift (Karlsruhe) [21]
2. LATEX
3. Human Readable TEX[22]

In theory, it is possible to write this way using both six or eight dots. In practice, however, these systems (and other similar ones) are combined with eight-dot coding of scientific symbols (see 3.1.3 above), thus presenting the syntactic frame for the mathematical writing, which in the six-dot systems is usually solved internally (i.e., encoded by the national braille standard).

3.3 Solutions for special typographic arrangement and graphics

As mentioned above, the difficulty in making visual documents accessible is not primarily determined by the number of specific marks, but mainly by the complexity of the typographic arrangement, i.e. by the number of the typesetting methods employed and by the extent of their visuality. With regard to traditional braille systems, the Instruction Manual for Braille Transcribing [23], published by National Library of Congress and containing 188 pages, remains a unique work. It is a proof of a very substantial effort to understand braille as a transcription of visual procedures and demonstrates how difficult (and often controversial from the methodological point of view) it is to require braille to copy all the differences that have been created in traditional typography for the needs of sighted readers.

The most difficult task concerning tactile notation remains pure graphics. Narrowing this very complex problem down to geometry, one can see that the significance of geometrical graphics as an integral part of mathematics has never diminished; on the contrary, following the multimedia wave at the end of the 20th century, it gained prominence and it can nowadays be done by dozens of applications. There might be a question of the usefulness of its transposition into tactile graphics (compared to the understanding of algebraic models and functions). Although it is obvious that the role of tactile graphics cannot be considered the same as that of visual graphics, there are several reasons why it is impossible to leave out graphics out of documents for the blind.

- some of the blind have acquired sight loss, and graphics is as necessary for their way of perceiving and thinking as it is for the sighted
- some of the people blind from birth have inherited visual perception and memory as well, and they have the same needs as the first group
- the rest of the blind would spontaneously do without graphics; yet their integration into the society of the sighted forces them to follow the visual issues, hence graphics is necessary for them as well

To sum up: Providing services for the blind at the university means to guarantee accessibility to an open set of diverse graphical symbols and mostly visual methods through an equally diverse and open set of tactile methods (or voice methods, which we have omitted for the sake of simplicity). This seems to be, at least at first sight, an unsolvable problem, and it is interesting to be reminded of technological procedures that aim at solving the unsolvable.

4 Applications designed to make mathematics accessible

The assertion of information technology and its shift from purely alphanumerical base towards multimedia in the last two decades enabled digitization of mathematical documents containing all the above-mentioned features. The amount of tools for typesetting and publishing of mathematical (and scientific in a broader sense) documents has grown and new formats have been defined for this purpose. It is however a well-known paradox that this development has only slightly changed the position of TeX as the dominant typesetting format for mathematicians (with PDF as the typical presenting format). In other formats, mathematical notation is usually inserted as graphical object. These techniques are in principle based on visual (presentation) form of a digitized notation. It is true that the number of documents using MathML for mathematical notation is increasing (as well as the number of systems with integrated MathML), but they mainly use the presentation variant of this format.

We do not consider as paradoxical the fact that presentation formats dominate in wide practice, although this might be a problem for the visually impaired. We are aware that a production of a mathematical document in a purely semantically oriented format tends to be less practical for at least two reasons:

- The exact semantics of a mathematical expression may not be clear when being formulated or solved, and in it does not need to be surely known to the typesetter. In common cases, even the author may know the sense of the notation rather intuitively; thus the visual form of the notation is easier to grasp.
- In certain (not always marginal) cases, the sense of a given mathematical symbol is new or at least shifted in comparison to how the standard meaning is.

4.1 Conversion between conventional formats and notations for the blind

The above analysis suggests that individual mathematical notations for the blind differ by the degree of attachment either to the visual notation (context oriented) or to the semantic (functional) aspect. Theoretically, the conversion between conventional, visually oriented formats and a tactile format could be more successful if the target tactile format is one of those which are visually oriented as well. Yet, this is not often confirmed in practice because it is not always the case that the same visual features make part of the source format and the target format, in spite of being of the same type. Further, for practical reasons the existing conversion tools only cover some areas of mathematical notation which may not be sufficient for university courses. In the case of conversion tools from the LaTeX format in particular, the degree of variability of this format often represents an obstacle. But if we are not expecting a fully automatic conversion, they can be certainly used for the initial conversion stage.

Technologies to be listed in this category:

- **LaBraDoor (LaTeX-to-Braille-Door)**; Johannes Kepler University Linz [28]
  Although the system was originally intended primarily as a module for a blind person's working environment with mathematical documents, it is mainly used for producing tactile mathematical literature for secondary and tertiary levels in the Marburg System and Human Readable TEX. The system is thus a conversion tool between the LATEXformat and the tactile mathematical notation, and vice versa.
- **latex-access**; Robin Williams, Alastair Irving [30]
  A system for a realtime translation of a line LATEXinto braille in the Nemeth codification.
- **Duxbury Braille Translator**, Duxbury Systems [31], [32]
  A complex system for converting documents into tactile notation in a number of braille notation systems - it translates the mathematical notation from the source document in some of the LaTeX format variants into the tactile notations Nemeth, British Mathematical Braille, and French Braille.
- **MAVIS**, Karshmer, Gupta [33]
  A pioneer project solved the translation from Nemeth code to LaTeX using language semantics and logic programming.
- **Insight project**, Annamalai, Gopal et al., Wang [34]
  It further improved upon the MAVIS project to develop a complete system for translating Math documents in Nemeth code with embedded text (in Grade II Braille) to LATEX. It also includes image processing to recognize the Braille dots to get an input for the translation.
- **BraMaNet** (Braille Mathématique sur InterNet), Mission Handicap [35]
  A system using the XSL stylesheets for the conversion of MathML into French Braille. In cooperation with the application MathType, it is able to translate a mathematical notation in a MS Word document. It is further a part of the NAT Braille project.
- **Math2Braille**, D. Crombie, R. Lenoir, et. al., FNB Netherlands (Dedicon) [36], [37]
  An open-source module to convert MathML to Dutch Math Braille notation.
- **Universal Math Conversion Library**, Archambault, Fitzpatrick, et. al., International Group Universal Math Accessibility [38], [39], [40], [41]
  Programming library encapsulating various converters for different Braille codes; it is based on a Presentation MathML (UMCL Canonical MathML). As output modules available are MathML to French, Italian, Marburg British and Nemeth Math codes, the system allows conversion from MathML to those codes. LaTeX to MathML and additionally Marburg code to MathML modules are available.
- **liblouisxml - Braille transcription software for XML documents** [43], [42]
  A library of liblouis - an open-source braille translator and back-translator. It enhances liblouis to convert XML (MathML) to any of dozens braille codes implemented in liblouis.
- **odt2braille**, Katholieke Universiteit Leuven [44]
  An extension to OpenOffice/LibreOffice (Writer) enabling to translate documents into various Braille codes including mathematical content which is translated into a braille math code (Nemeth code, British code, Marburg system and Woluwe code). It is powered by liblouisxml.
- **odt2daisy**, SUN Microsystems, Katholieke Universiteit Leuven [46]
  OpenOffice/LibreOffice extension enabling to export a document in Daisy 3 format (Specification for the Digital Talking Book Modular Extension for Mathematics) including mathematical content conforming to the MathML standard. It is powered by liblouisxml.

- **PEF - Portable Embosser Format**, Westling, Håkansson [47]
  It is not a conversion tool but data format for representing Braille documents independently of language, braille code, embosser settings or computer environment. It could be used for braille embossing or archiving. It is listed here in this context as some of the conversion tools above utilize PEF format for the resulting documents (e.g. odt2braille).

We suppose that tools and systems of this category will be utilized especially by those who provide authoring of documents in a format accessible to the blind (service providers, sighted collaborators, etc.). Considering the amount of the tools available (the list is not complete, of course) and considering the fact that their aim and usability in different cases of practical situations are different, it is always necessary to combine them in practice and to analyze the source material. In these circumstances it is hardly conceivable that those tasks (such as analysis of the source document, all know-how in details, combining the tools) are practically feasible for a blind user. In spite of the facts mentioned, it is known that these conversions of conventional documents are often viewed as a blind user's responsibility.

4.2 Tools creating accessible documents with mathematical formulae

While the technologies of the first category are intended to convert existing documents, in the next group of technologies, systems used for creating documents are listed. These systems always include some conversion algorithms and thus the two categories cannot be distinguished precisely. Many of the technologies are built accessible and usable for blind users and in case a blind user is responsible for authoring documents, the amount and variety of technologies he/she can use is increasing (see above).

Technologies to be mentioned here:

- **InftyReader, InftyEditor**, InftyProject organization [49], [50]
  While InftyReader recognizes scanned images of printed documents including math formulae, InftyEditor is an authoring tool for mathematical documents. They can produce output in various formats such as IML (Infty XML), LaTeX, MathML, Daisy with MathML, Human Readable TeX, PDF, MS Word 2007 etc.
- **MathType**, Design Science [51]
  Math equation editor (an enhanced version of the editor embedded in MS Word) that produces output in MathML.
- **MathDaisy**, Design Science c, [60], [62]
  Together with MS Word and MathType, MathDaisy provides translation of mathematical formulae included in a MS Word document into Daisy 3 format (Specification for the Digital Talking Book Modular Extension for Mathematics).
- **Lambda - Linear Access to Mathematic for Braille Device and Audio-synthesis**, Lambda Project, Veia progetti [20]
  Although primarily Lambda is a system for writing, manipulation and viewing of mathematical expressions for the blind and their sighted collaborators, it could be used for authoring more complex mathematical documents since Lambda can create documents in XHTML+MathML format.
- **Index WinBraille with Math**, Index Braille [52]
  A system authoring braille documents to be embossed by Index Braille embossers. The system is able to translate math expressions embedded in MS Word documents (as Equation editor) to Nemeth code and Swedish math notation.
- **Duxbury Braille Translator**, Duxbury Systems [31], [32]
  see above
- **Tiger Software Suite**, ViewPlus Technologies [53], [54]
  A complex system for authoring braille documents including tactile graphics and mathematical expressions which are processed in conjunction with MathType editor. It is able to translate mathematics to Nemeth as well as British and French braille math codes.
- **BUF – Braille Universal Format**, Masaryk University Brno [48]
  A complex set of MS Word macros which gives a potential of authoring braille documents including mathematical expressions (Czech braille math code only). The system has been used for producing braille scientific documents by Masaryk University for 10 years.

4.3 Working environment for both blind and sighted collaborators

So far we have only considered the task to make a mathematical document accessible and supposed that the document will be read. But the academic practice involves structured browsing through the document, writing and producing formulae not for the presentation but as a part of solving some issue ("doing math"). Technologies of the last category help to deal with these situations and operations. Nevertheless, it is not possible to say that those technologies solve all ranges of mathematical practice and make them accessible because in our entire analysis of technologies we have intentionally considered only those mathematical documents and operations that are based on algebraic content. We have left aside the fact that the content of mathematical documents (as well as scientific ones in a broader sense) very often includes graphics (geometry) and that their creation is presupposed.

Technologies to be mentioned:

- **ChattyInfty**, InftyProject organization [49], [50]
  It is an extended version of InftyEditor enabling to provide speech output of editing data of InftyEditor, including math expressions. Thus it enables writing and manipulation of mathematical expressions in documents.
- **Lambda - Linear Access to Mathematics for Braille Device and Audio-synthesis**, Lambda Project, Veia progetti [20]
  An integrated system (primarily editor) enabling to manipulate mathematical documents and providing both braille and speech output. It is unique as it is based on its own internal code which is independent of language or braille code. For document braille presentation, the internal code is transformed into one of the national 8-dot braille codes which have been developed for the purposes of Lambda. The visual presentation of the document on-screen is linear but still accessible for the sighted; the document can be displayed in the conventional math notation on demand.
- **BlindMoose**, Masaryk University Brno [55]
  A complex set of MS Word macros which enables to read, edit and manipulate mathematical expressions in the user interface of MS Word. The document is displayed on-screen linearized yet accessible to sighted collaborators. Document in Braille is presented in Czech Braille Code only; BlindMoose provides no speech output.
- **WinTriangle**, Gardner [57], [56]
  A specialized RTF word processor capable of displaying and voicing documents including mathematical formulae which are displayed in a linear form (accessible to sighted collaborators).
- **MaWEn - Mathematical Working Environment**, Miesenberger, Stöger, Batušić et al. [58]
  One of the systems of mathematical working environment which systematically consider extended support of navigation in formula structure, text selection and other manipulation tasks. Theoretical concept.

Particular technologies enabling reading of mathematical documents:

- **Daisy 3**, Daisy Consortium [59], [60], [61], [62]
  Documents in Daisy 3 format (DAISY/NISO Z39.86 Specifications for the Digital Talking Book) can embed mathematical expressions in MathML according to DAISY 3 Modular Extension for MathML. Practically, those documents must be read by Daisy reader software which supports MathML in Daisy (e.g. ReadHear PC Premium or ChattyInfty).
- **MathPlayer**, Design Science [63]
  A web browser plugin for the conventional visualization of MathML content embedded in XHTML documents. It provides speech output according to rules based on lexical cues and enables users to navigate in the structure of a mathematical expression (tree-based navigation).

5 Conclusion

Since several years, Masaryk University has been integrating nearly one hundred visually impaired students in its study programmes and has been running a nationwide online library of scientific documents[13] made accessible for other universities and schools in the Czech Republic and Central Europe in general.

[13] University Library for Students with Special Needs. Teiresias Centre, Masaryk University: Brno, Czech Republic.

We have our own applications (see above) and try to use those produced by other teams, considering very carefully the existing tools which may help people with visual impairment, both students and teachers, to do mathematics. In practice, however, we still have to do much of the work manually due to the lack of compatibility and universality of so many of the applications available.

Two thousand years ago the first mathematical books were published in Europe, and we have seen that the issues they deal with are remarkable in spite of the lack of specific tools that were only developed later. Two thousand years later, it is not rare that our blind students are restricted in the same way, because the technical tools cannot be used, for one or another reason. The task of solving the unsolvable has not been accomplished yet. Last but not least, developing applications for conversion between formats is a popular exercise among IT-oriented scholars, who, however, do not always set the goal to create a functional tool ready for practical purpose, compatible with other tools and easy to develop further; this is the reason why today many promising projects remain without any further development.

What can be done to overcome the difficulties? As in many other cases, the point is not so much in changing the behavior of the blind, but in the correct behavior of the sighted. The unsolvable problem can be solved and, paradoxically, it is more or less clear how to do it: by using unambiguous standard format as an intermediary. Give me the place to stand, and I shall move the earth. Then any newly created document will be converted into this format for archivation or further conversion, and tools for processing it can be developed for national, regional, local and individual purpose. We can say we are not so far from achieving that aim: there are encoding tools for both visual symbols and braille symbols, and the intermediary standards has been proposed (could be MathML, XHTML, Unicode etc.). What remains to be done is just the most difficult task: to make these intermediaries unambiguous and to respect them.

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